The acronym "NOTA" stands for "None Of The Above". You may find this information useful:
The standard form of a conic centered at the (h, k) is given by:
$a(x-h)^2 + b(x-h)(y-k) + c(y-k)^2$=1 which expands to $-1 + ah^2 + bhk + ck^2 - 2ahx - bkx + ax^2 - bhy - 2cky + bxy + cy^2 = 0$ and if the conic is an ellipse, its enclosed area is given by $\frac{2\pi i}{\sqrt{b^2-4ac}}$ where $i = \sqrt{-1}$.

1. The conic $x^2 + 4x + y^2 - 16y = 0$ has its center located at (h, k) and has an enclosed area of $A\pi$ units. What is $A + h + k$?

A. 62 B. 74 C. 22 D. $68\pi + 6$ E. NOTA

2. What is the distance between the lines $3x + 2y = 5$ and $6x + 4y = 20$?

A. $\frac{5}{\sqrt{2}}$ B. $\frac{5}{\sqrt{5}}$ C. $\frac{5}{\sqrt{13}}$ D. 5 E. NOTA

3. One of the asymptotes of the hyperbola $16x^2 - 64x - 9y^2 + 54y = 161$ can be written as $x + by + c = 0$ where $b > 0$. What is $\frac{b}{c}$?

A. $-\frac{17}{3}$ B. $-\frac{16}{51}$ C. $-\frac{3}{17}$ D. $-\frac{51}{16}$ E. NOTA

4. The directrices of the ellipse $3x^2 - 6x + y^2 - 16y + 58 = 0$ are $y = p$ and $y = q$. What is the product pq?

A. $\frac{229}{4}$ B. $\frac{283}{4}$ C. $\frac{101}{2}$ D. $\frac{155}{2}$ E. NOTA

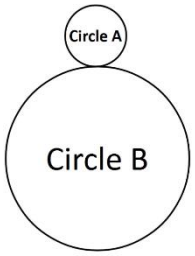
5. For the parabola given by $y^2 + 8y - 6x + 4 = 0$, the length of the latus rectum is a and the focus is located at (b, c). What is abc?

A. -12 B. 12 C. -30 D. 30 E. NOTA

6. There is a rectangle inscribed in the ellipse $3x^2 - 6x + y^2 - 16y + 58 = 0$. The latera recta make up two edges of the rectangle. Find the positive difference between the areas enclosed by the rectangle and by the ellipse.

A. $27\pi - 8\sqrt{6}$ B. $3\sqrt{3}\pi - 4\sqrt{6}$ C. $3\sqrt{3}\pi - 2\sqrt{6}$ D. $6\sqrt{3}\pi - 2\sqrt{6}$ E. NOTA

7. The radius of circle B is 2018 times the radius of circle A. Circle A rolls around circle B, always maintaining a common tangency point, on a single trip ending at its starting position. Using the background in the diagram shown below as the point of reference, how many times will the letters "circle A" be upright and parallel to the bottom of this page, including this first one?



A. $\frac{2017}{2}$ B. 1009 C. 2017 D. 2018 E. NOTA

8. How many of the following statements are true regarding conic sections?

   I. Eccentricity can be negative.

   II. A rectangular hyperbola (hyperbola with perpendicular asymptotes) always has an eccentricity of 2.

   III. A non-degenerate parabola can be a function.

   IV. A non-degenerate ellipse can be a function.

   V. A non-degenerate hyperbola can be a function.

A. 1 B. 2 C. 3 D. 4 E. NOTA

9. There are 2018 points evenly spaced out on a circle. Three distinct points are chosen at random. The probability that the points form a right triangle is $\frac{a}{b}$ where a and b are relatively prime positive integers. What is $a + b$?

A. 2020 B. 2019 C. 2018 D. 2017 E. NOTA

10. There are 2018 points evenly spaced out on a circle. Three distinct points are chosen at random. The probability that the points form an acute triangle is $\frac{a}{b}$ where a and b are relatively prime positive integers. What is $a + b$?

A. 3 B. 2019 C. 5041 D. 7055 E. NOTA

11. The hyperbola $2x^2 + \sqrt{\pi}xy - y^2 = 1$ has directrices given by $y = ax + b$ and $y = cx + d$. What is $a + b - c + d$? (Hint: refer to the useful information on the front of this test.)

A. $2 - \pi$ B. $2\sqrt{\pi} + \frac{\pi}{2}$ C. $2\sqrt{\pi} - \frac{\pi}{2}$ D. $2 + \pi$ E. NOTA

For questions 12-13 refer to the following: A conic section contains the point $(1,1)$ and has a focus at $(0,0)$ with its corresponding directrix with equation $x + y = 6$.

12. What is the eccentricity of the conic section?

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{1}{\sqrt{2}}$ D. 2 E. NOTA

13. The equation of the conic section is given as $Ax^2 - 2xy + Cy^2 + Dx + Ey + F = 0$.

What is $A + C + D + E + F$?

A. 4 B. 2 C. 0 D. -2 E. NOTA

14. Rank the eccentricities of the following graphs from greatest to least.

I. $x^2 + y^2 = 2018$

II. $4x^2 - 16y^2 = 64$

III. $2018!\,(x - 2018)^2 + 2018!\,(y - 2018)^2 = 2018!\,y^2 + 2018!$

IV. $xy = 1$

V. $3x\text{^}2 + y\text{^}2 = 2018$

A. I, IV, II, III, V B. IV, II, III, V, I C. II, IV, III, V, I D. I, II, IV, III, V E. NOTA

15. Find the area of the region bounded by the x −axis and the graph of $y \leq \sqrt{9 - 4x^2}$

A. 2π B. $\frac{81\pi}{8}$ C. $\frac{9\pi}{4}$ D. $\frac{9\pi}{2}$ E. NOTA

16. What is the length of the latus rectum of $\frac{x^2}{4} - \frac{y^2}{16} = 1$?

A. 32 B. 16 C. 8 D. 4 E. NOTA

17. A rectangle is inscribed in the region bounded by $y \geq -3x + 1, y \geq x - 3, y \leq 0$. If one edge of the rectangle lies on the x −axis, what is the area of the largest possible rectangle?

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{4}{3}$ D. $\frac{8}{3}$ E. NOTA

18. The equation of the conic section $2x^2 + 6y^2 + 2y + 8x - 4 = 0$ is rewritten into the product of three matricies: $[x \quad y \quad 1] \cdot \begin{bmatrix} a_1 & a_2 & a_3 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = [0]$. What is the determinant of the middle matrix on the left side of this equation?

A. -146 B. -98 C. 96 D. 144 E. NOTA

19. A hyperbola with the equation $\frac{(x+2)^2}{16} - \frac{(y-1)^2}{20} = 1$ has a latus rectum contained in Quadrants II & III. That latus rectum is the same latus rectum as that of a parabola. What is the product of all the possible x-intercepts of the parabola?

A. $\frac{756}{25}$ B. $\frac{1431}{25}$ C. $\frac{1456}{25}$ D. $\frac{1484}{25}$ E. NOTA

20. The asymptotes of the hyperbola given by $-x^2 + 4xy - 3y^2 - 3x + 7y + 10 = 0$ are $y = mx + b$ and $y = nx + c$. What is $bm + cn$?

A. $\frac{19}{9}$ B. $\frac{8}{9}$ C. $-\frac{51}{9}$ D. $-\frac{25}{9}$ E. NOTA

For questions 21-22, refer to the ellipse given by $x^2 + xy + y^2 + 2x - 8y + 4 = 0$

21. What is the center of the ellipse?

A. $(-1,4)$ B. $(1,4)$ C. $(3,-2)$ D. $(-4,6)$ E. NOTA

22. What is the area enclosed by the ellipse?

A. $16\pi\sqrt{3}$ B. $8\pi\sqrt{3}$ C. $\frac{8\pi\sqrt{3}}{3}$ D. $\frac{2\pi\sqrt{3}}{3}$ E. NOTA

23. A parabola whose vertex is the point $V(-1,1)$ and focus is $(3,3)$ has equation $ax^2 + bxy + cy^2 + dx + ey + f = 0$, where the greatest common divisor between the coefficients of the equation is 1. Find $|a + b + c + d + e + f|$.

A. 0 B. 54 C. 156 D. 178 E. NOTA

24. Given the ellipse $3x^2 - 12x + 2y^2 + 12y + 6 = 0$, there exists a real simplified number $k = a\sqrt{b} + c$, (where a, b, c are all positive integers and b is not divisible by the square of any integer) such that the hyperbola $xy - 2y + 3x = k$ is tangent to the ellipse at two points. Find abc.

A. 72 B. 48 C. 36 D. 24 E. NOTA

25. The circle that contains the points $A(1,1), B(3,9), C(-1,5)$ has equation $(x-h)^2 + (y-k)^2 = r^2$.

Find $hk + r^2$.

A. $\frac{10}{3}$ B. $\frac{300}{9}$ C. $\frac{310}{9}$ D. $\frac{590}{9}$ E. NOTA

For questions 26-27: There exists a locus of points in a plane for which the sum of the distances from (5,12) and some unknown point (p, q) is 20. Furthermore, the point (0,0) is included amongst the locus of points.

26. Determine the largest area enclosed by the different possible loci of points.

A. 49π B. $10\pi\sqrt{91}$ C. 80π D. ∞ E. NOTA

27. Determine the smallest area enclosed by the different possible locus of points.

A. 7π B. $\pi\sqrt{91}$ C. 8π D. π E. NOTA

28. For point $P(x_1, y_1)$ and line m with equation $ax + by + c = 0$, let point $Q(x', y')$ be the reflection of $P(x_1, y_1)$ across line m. Which of the following gives the values of (x', y') in terms of a, b, c, x_1, y_1?

A. $\left(x_1 + \frac{a(ax_1+by_1+c)}{a^2+b^2}, y_1 + \frac{b(ax_1+by_1+c)}{a^2+b^2}\right)$

B. $\left(x_1 + \frac{2a(ax_1+by_1+c)}{a^2+b^2}, y_1 + \frac{2b(ax_1+by_1+c)}{a^2+b^2}\right)$

C. $\left(x_1 - \frac{a(ax_1+by_1+c)}{a^2+b^2}, y_1 - \frac{b(ax_1+by_1+c)}{a^2+b^2}\right)$

D. $\left(x_1 - \frac{2a(ax_1+by_1+c)}{a^2+b^2}, y_1 - \frac{2b(ax_1+by_1+c)}{a^2+b^2}\right)$

E. NOTA

29. Let C be the graph of $xy + x = 1$, and denote by C' the reflection of C in the line $y = 2x$. Let the equation of C' be written in the form $ax^2 + bxy + cy^2 + dx + ey + f = 0$, where the greatest common divisor between the coefficients of the equation is 1. Find $|a + b + c + d + e + f|$.

A. 7 B. 11 C. 13 D. 17 E. NOTA

30. What is the distance from the point $(2018, -2)$ to $x^2 + 2y^2 + 2x + 8y + 9 = 0$?

A. 2020 B. 2019 C. 2018 D. 2017 E. NOTA